

# $SO(10)$ and Large $\nu_\mu - \nu_\tau$ Mixing

**S.M. Barr**

Bartol Research Institute, Univ. of Delaware  
Newark, DE 19716

and

**C.H. Albright**

Fermi National Accelerator Laboratory  
Batavia, IL 60510

February 1, 2008

## **Abstract**

A general approach to understanding the large mixing seen in atmospheric neutrinos is explained, as well as a highly predictive  $SO(10)$  model which implements this approach. It is also seen how bimaximal mixing naturally arises in this scheme.

*Talk given at NNN99, SUNY Stony Brook, Sept. 22-24, 1999*

The problem of neutrino mixing should not be looked at in isolation, but as part of the larger “fermion mass problem”, i.e. the problem of understanding the masses and mixings of the quarks and leptons. The fermion mass problem is very old; serious efforts at model building go back more than twenty-five years, and in that time hundreds of models of quark and lepton masses have been published. However, we are now in a new and more hopeful

situation, for neutrino oscillations give us precious new clues in the search for the right theory as well as new ways of testing theories experimentally.

The most significant new clue, perhaps, is the largeness of the mixing of  $\nu_\mu$  (presumably with  $\nu_\tau$ ) seen at SuperK. What is this clue telling us? I will explain one idea for what it might be telling us which is based on certain features of grand unification. I will then mention and very briefly discuss a concrete model that incorporates this idea. (Actually, the model came first, and only then was it noticed that the model contains an attractive explanation of the large  $\nu_\mu - \nu_\tau$  mixing!) At the end I will address the question whether large  $\nu_e$  mixing is compatible with  $SO(10)$  in a simple way. The answer is yes.

Large  $\nu_\mu - \nu_\tau$  mixing came as a surprise. The puzzle is that the mixing of the second and third families is small for the quarks ( $V_{cb} \cong 0.04$ ) and large for the leptons ( $U_{\mu 3} \cong 1/\sqrt{2} \cong 0.7$ ). This is puzzling because both grand unification and flavor symmetry, the two most promising ideas for explaining fermion masses, tend to relate the quark and lepton parameters. Actually, there are *two* puzzles: the mixing of the second and third families is *too small for the quarks* and *too big for the leptons*, in a sense that I will now explain.

Many models are based on the old idea of Weinberg, Wilczek and Zee, and Fritzsch.<sup>1</sup> Looking at only the first two families, in the late 1970's they posited simple textures of the form

$$\overline{u_{iR}} U_{ij} u_{jL} = (\overline{u_R}, \overline{c_R}) \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} \begin{pmatrix} u_L \\ c_L \end{pmatrix} \quad (1)$$

and

$$\overline{d_{iR}} D_{ij} d_{jL} = (\overline{d_R}, \overline{s_R}) \begin{pmatrix} 0 & a' \\ a' & b' \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}. \quad (2)$$

The crucial features of these matrices are that they are *hierarchical* (i.e.  $a \ll b$  and  $a' \ll b'$ ) and *symmetric*. The eigenvalues of the down quark matrix are  $m_s \cong b'$ , and  $m_d \cong -a'^2/b'$ , and the rotation angle needed to diagonalize it is  $\theta_{ds} \cong a'/b' \cong \sqrt{m_d/m_s}$ , with analogous formulas for the up quarks. Thus the Cabibbo angle is given by

$$V_{us} \cong \sqrt{m_d/m_s} - e^{i\phi_{12}} \sqrt{m_u/m_c}. \quad (3)$$

Since  $V_{us} \cong 0.21$ ,  $\sqrt{m_d/m_s} \cong 0.21$ , and  $\sqrt{m_u/m_c} \cong 0.07$ , this relation works well for  $\phi_{12} \sim \pi/2$ . If one chooses analogous hierarchical and symmetric forms for the lepton mass matrices one obtains

$$U_{e2} \cong \sqrt{m_e/m_\mu} - e^{i\phi'_{12}} \sqrt{m_{\nu_e}/m_{\nu_\mu}}. \quad (4)$$

This also can work reasonably well if the small angle MSW solution to the solar neutrino problem is correct, as then  $U_{e2} \sim 0.04$ , and  $\sqrt{m_e/m_\mu} \cong 0.07$ .

If one supposes that the full  $3 \times 3$  mass matrices are symmetrical and hierarchical, one obtains similar predictions for the mixing of the second and third families, For example, extending the Weinberg-Wilczek-Zee-Fritzsch pattern to the third family, as Fritzsch did,<sup>2</sup> one finds

$$V_{cb} \cong \sqrt{m_s/m_b} - e^{i\phi_{23}} \sqrt{m_c/m_t}, \quad (5)$$

and

$$U_{\mu 3} \cong \sqrt{m_\mu/m_\tau} - e^{i\phi'_{23}} \sqrt{m_{\nu_\mu}/m_{\nu_\tau}}. \quad (6)$$

Since  $\sqrt{m_s/m_b} \cong 0.14$ , and  $\sqrt{m_c/m_t} \cong 0.04$ , one sees that the experimental value of the quark mixing  $V_{cb}$  ( $\cong 0.04$ ) is about a factor of 3 smaller than the Fritzschian expectation. On the other hand, since  $\sqrt{m_\mu/m_\tau} \cong 0.24$ , and  $\sqrt{m_{\nu_\mu}/m_{\nu_\tau}}$  may be presumed to be small, one sees that the experimental value of the lepton mixing  $U_{\mu 3}$  ( $\cong 0.7$ ) is about a factor of 3 larger than the Fritzschian expectation.

What we have argued in several papers<sup>3,4,5</sup> is that the trouble with such Fritzschian textures when applied to the heavier two families is that they are based on symmetric matrices. Let us see what happens if there are instead highly asymmetric or, as we have called them, “lopsided” textures. Consider a toy example with matrices

$$\overline{d_{iR}} D_{ij} d_{jL} = m(\overline{d_R}, \overline{s_R}, \overline{b_R}) \begin{pmatrix} - & - & - \\ - & 0 & \sigma \\ - & \epsilon & 1 \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \quad (7)$$

$$\overline{\ell_{iR}} L_{ij} \ell_{jL} = m(\overline{e_R}, \overline{\mu_R}, \overline{\tau_R}) \begin{pmatrix} - & - & - \\ - & 0 & \epsilon \\ - & \sigma & 1 \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \quad (8)$$

where  $\epsilon \ll \sigma \sim 1$ . We are interested at the moment in the heavier two families, so we do not write the entries of the first row and column, which are assumed to be small. The important feature of these matrices is the large, lopsided off-diagonal entry ( $\sigma$ ). For the mass matrices of the up quarks and neutrinos we assume that there is no such large off-diagonal element. Note another very important feature of these matrices, which is that the charged lepton mass matrix  $L$  is the *transpose* of the down quark mass matrix  $D$ .  $L = D^T$  is a “minimal  $SU(5)$ ” relation. The realistic model we will discuss later is based on  $SO(10)$ , nevertheless the  $SU(5)$  subgroup of  $SO(10)$  in that model relates  $L$  to  $D^T$ . The reason  $L$  is related to  $D^T$  is simple:  $SU(5)$  unifies the  $\ell_L^-$  with the  $d_R$  in  $\mathbf{\bar{5}}$  multiplets and the  $\ell_R^-$  with the  $d_L$  in  $\mathbf{10}$  multiplets. Thus the mass matrix of the charged leptons is related to that of the down quarks only up to a left-right transposition. As we shall see, this feature allows a simple explanation of the double puzzle of  $U_{\mu 3}$  and  $V_{cb}$ .

The point is that the observed mixings are of *left-handed* fermions. Specifically, we may write  $U_{\mu 3} \cong \theta_{\mu\tau}^{left} - \theta_{\nu_\mu\nu_\tau}^{left}$ , and  $V_{cb} \cong \theta_{sb}^{left} - \theta_{ct}^{left}$ . Thus  $V_{cb}$  and  $U_{\mu 3}$  are really not directly related to each other by  $SU(5)$ ; rather each is related to some *right-handed* mixing angle. Specifically,  $SU(5)$  relates  $\theta_{sb}^{left}$  to  $\theta_{\mu\tau}^{right}$ , and in the toy example both are of order  $\epsilon$ , as Eqs. (7) and (8) show. And  $SU(5)$  relates  $\theta_{\mu\tau}^{left}$  to  $\theta_{sb}^{right}$ , and in the toy example both are of order  $\sigma$ . Moreover, from the form of the matrices in Eqs. (7) and (8) one sees that the ratio of masses of the fermions of the second and third families are given by  $m_2/m_3 \sim \sigma\epsilon$ . Thus, given the experimental values of the quark and lepton mass ratios, the values of  $\epsilon$  and  $\sigma$  are inversely related, so that the smallness of  $V_{cb}$  and the largeness of  $U_{\mu 3}$  are two sides of the same coin. The expectation from assuming symmetric forms, as in the Fritzschian textures, for example, would be that the mixing angles  $\theta_{23}^{Fritzschian}$  are of order  $\sqrt{m_2/m_3}$ , which as we just have seen is to say that they are of order  $\sqrt{\sigma\epsilon}$ . However, the form of Eqs. (7) and (8) tells us that actually  $V_{cb} \sim \epsilon$  and  $U_{\mu 3} \sim \sigma$ . Thus, as observed,  $V_{cb}/\theta_{23}^{Fritzschian} \sim \theta_{23}^{Fritzschian}/U_{\mu 3} \sim \sqrt{\epsilon/\sigma} < 1$ .

A striking fact about the mechanism just described is that the large mixing seen in atmospheric neutrino data is coming from a large off-diagonal entry in  $L$ , the mass matrix of the *charged* leptons. One is accustomed to speak of “neutrino mixing angles”, but this is obviously a misnomer, since the leptonic angles are really the mismatch between the charged and neutral leptons, just as the KM angles are the mismatch between the up and

down quarks. The mechanism I have just described has three ingredients: (1) There is large mixing in  $L$ ; (2)  $L$  is highly asymmetric in its 23 block; and (3)  $L$  is related by  $SU(5)$  to the transpose of  $D$ .

Although I have described it in a toy example based on  $SU(5)$ , this mechanism first emerged independently in several models that used various groups.<sup>3,4,5,6,7</sup> The paper of Sato and Yanagida<sup>6</sup> used the group  $E_7$  broken down to an  $SU(5) \times U(1)$  subgroup. The paper of Irges, Lavignac, and Ramond used  $SU(3) \times SU(2) \times U(1)$ <sup>3</sup>, where anomaly cancellation led to  $SU(5)$ -like conditions on the mass matrices. Our papers<sup>3,4,5</sup> were based on  $SO(10)$ , which, of course, has  $SU(5)$  as a subgroup. Since these first papers, many papers<sup>8</sup> have appeared that consider this idea in models with the unified group  $SU(5)$ , while an  $SO(10)$  model quite similar in some respects to the one I am about to discuss has been proposed by Babu, Pati and Wilczek.<sup>9</sup> In a short talk I cannot go into the details of our model. The point I wish to emphasize is that this model was first constructed without any thought to the pattern of neutrino masses and mixings that would arise. The goal in our first paper<sup>3</sup> was rather to construct a realistic  $SO(10)$  model with as simple a Higgs structure as possible. The “minimal Higgs structure” in  $SO(10)$  involves the use of only the following non-singlet Higgs representations to break  $SO(10)$  down to the standard model group  $G_{SM}$ :  $\mathbf{45}_H + \mathbf{16}_H + \overline{\mathbf{16}}_H$ . That the breaking down to  $G_{SM}$  can be done with these fields was shown in Ref. 10. Starting with this minimal Higgs structure for  $SO(10)$  breaking, we looked for the simplest effective Yukawa operators that can reproduce the pattern of quark and lepton masses that is seen. By “simple” operators we mean operators of low dimension, which can arise from simple tree-level diagrams where small multiplets are integrated out.

We were led, practically uniquely, to a set of six Yukawa operators that give the following Dirac mass matrices for the up quarks, down quarks, neutrinos, and charged leptons at the GUT scale.<sup>3,4</sup> (The Majorana mass matrix  $M_R$  of the right-handed neutrinos comes from different terms.)

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m_U, \quad (9)$$

$$D = \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{pmatrix} m_D, \quad (10)$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m_U, \quad (11)$$

$$L = \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma + \epsilon & 1 \end{pmatrix} m_D. \quad (12)$$

Much about the pattern of entries in these matrices can be understood in purely group-theoretic terms. The “1” entries come from the usual minimal Yukawa term  $\mathbf{16}_3 \mathbf{16}_3 \mathbf{10}_H$ . As is well-known, such a term gives equal contributions to the up quark matrix  $U$  and neutrino Dirac matrix  $N$ , and it also gives equal contributions to  $D$  and  $L$ . That is why we can write the matrices in terms of only two overall scales  $m_U$  and  $m_D$ . The “ $\epsilon$ ” entries arise from the lowest dimension Yukawa operator that involves the  $\mathbf{45}_H$ , namely  $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \mathbf{45}_H$ . Because the vacuum expectation value of the  $\mathbf{45}_H$  must be proportional to the generator  $B - L$  in order to do the breaking of  $SO(10)$  (in particular the doublet-triplet splitting), there arises from this term a relative factor of  $-1/3$  between the quark matrices and the lepton matrices, which can be seen in Eqs. (9) — (12). It can also be shown that with  $\langle \mathbf{45}_H \rangle \propto B - L$  the quartic Yukawa operator involving this VEV gives a flavor-antisymmetric contribution, as also seen in Eqs. (9) — (12). The lop-sided  $\sigma$  entries come from a quartic term  $\mathbf{16}_2 \mathbf{16}_3 \mathbf{16}'_H \mathbf{16}_H$ , where the  $SO(10)$  indices are contracted in a certain way. (No effective Yukawa of lower dimension can be written down that involves the  $\mathbf{16}_H$ .) It is well-known that such a four-16 operator contributes only to the down quark and charged lepton mass matrices. Moreover, since the expectation value of the spinor  $\mathbf{16}_H$  breaks  $SO(10)$  only down to  $SU(5)$ , this operator respects the minimal  $SU(5)$ -relation  $L = D^T$ . The entries  $\delta$  and  $\delta'$  also arise from four-16 operators, though ones where the  $SO(10)$  indices are contracted differently in a way that gives flavor-symmetric contributions. However, for the same reason as in the case of  $\sigma$ , these only appear in  $D$  and  $L$ .

One cannot in a short talk explain in detail the structure of the model. Suffice it to say that the mass matrices arise from very simple Yukawa struc-

tures that in turn arise from very simple particle content. The model is thus not only simple at the level of the mass matrix “textures”, but also at the level of the underlying unified model.

Although it has very few parameters, this model gives a remarkably good fit to all the quark and lepton masses and mixings.<sup>4</sup> There are altogether *nine* predictions. Three of them are well-known relations that arise in many models because of the group theory of  $SU(5)$  and  $SO(10)$ : **(1)**  $m_b^0 \cong m_\tau^0$  (the superscript zero refers throughout to quantities evaluated at the unification scale); **(2)**  $m_s^0 \cong \frac{1}{3}m_\mu^0$ ; **(3)**  $m_d^0 \cong 3m_e^0$  (these last two relations are the well-known Georgi-Jarlskog relations).

The fourth prediction is that  $m_u$  is relatively small. **(4)**  $m_u/m_t \ll m_d/m_b, m_e/m_\tau$ . The point is that the Yukawa terms (involving the parameters  $\delta$  and  $\delta'$ ) that generate masses for the other fermions of the first family ( $d$  and  $e$ ) leave the  $u$  quark massless. (We have seen the group-theoretical reason for this.) This accords well with the fact that  $m_d^0/m_b^0 \cong 10^{-3}$ , and  $m_e^0/m_\tau^0 \cong 0.3 \times 10^{-3}$ , whereas (assuming  $m_u \approx 4$  MeV) the comparable ratio  $m_u^0/m_t^0$  is only about  $0.6 \times 10^{-5}$ . Thus a small Yukawa term ( $\eta$ ) must be introduced in this model to give a non-vanishing mass for the  $u$  quark. This  $\eta$  term has interesting consequences for neutrino masses, as we shall see.

A remarkable postdiction of the model is the charm quark mass<sup>4</sup>: **(5)**  $m_c(m_c) = (1.1 \pm 0.1)$  GeV. This is remarkable for two reasons. First, the charm quark mass is generally a severe problem for  $SO(10)$ , since the simplest  $SO(10)$  schemes predict that  $m_c^0/m_t^0 = m_s^0/m_b^0$ , which is an order of magnitude too large. Second, this model not only gives a postdiction of  $m_c$  that is of the right order of magnitude, but even predicts it correctly to within about 15% accuracy, which is quite acceptable given the uncertainties.

Another remarkable success of the model is a prediction of  $V_{ub}$ : **(6)**  $V_{ub} = 0.0052e^{i\theta} - 0.0028$ . The phase  $e^{i\theta}$  is the single non-trivial physical phase angle contained in the parameters appearing in Eqs. (9) — (12). This phase is not fixed by other measured masses or mixings. So the prediction of the model is that  $V_{ub}$  lies on a certain circle in the complex plane. As it happens, this circle slices precisely through the middle of the presently allowed region in the  $(Re(V_{ub}), Im(V_{ub}))$  plane.

Finally, there are predictions for the three neutrino mixing angles. **(7)** The most significant prediction is that the  $\nu_\mu - \nu_\tau$  mixing is large. This stems from the large entry  $\sigma$  in  $L$ . A fit to the known quark and lepton masses and mixings gives  $\sigma \cong 1.8$ ,  $\epsilon \cong 0.14$ ,  $\delta \cong 0.008$ ,  $|\delta'| \cong 0.008$ , and

$\eta \cong 0.6 \times 10^{-5}$ . Thus the angle  $\theta_{\mu\tau}^{left}$  appearing in  $U_{\mu 3} = \sin(\theta_{\mu\tau}^{left} - \theta_{\nu_\mu\nu_\tau}^{left})$  is given by  $\theta_{\mu\tau}^{left} \cong \tan^{-1} \sigma \cong \pi/3$ . The angle  $\theta_{\nu_\mu\nu_\tau}^{left}$  is not exactly predicted by the model, since it depends on the unknown Majorana mass matrix  $M_R$  of the right-handed neutrinos. But we can say from the form of Eq. (11) that it is of order  $\epsilon$  and thus small.

(8) The prediction for the mixing of the electron neutrino is quite interesting because, depending on what one assumes for  $M_R$ , it comes out quite naturally to give *either* the small-angle MSW solution to the solar neutrino problem *or* the vacuum “just-so” solution. If one supposes that the matrix  $M_R$  does not have large mixing between the first family and the other families, i.e. that the 12, 21, 13, and 31 elements of  $M_R$  are negligible, then the mass matrix of the light neutrinos, which has the usual “see-saw” form  $M_\nu = -N^T M_R^{-1} N$ , also gives very little mixing between the first family and the others. (See Eq. (11).) This is also the case no matter what the form of  $M_R$  if the parameter  $\eta = 0$  (meaning that  $m_u = 0$ ). In these cases, the mixing of the electron neutrino comes entirely, or almost entirely, from diagonalizing  $L$ . In that case one gets a sharp prediction that  $\sin^2 2\theta_{e\mu} \cong 16 \times 10^{-3} \cos^2 \theta_{\mu\tau}$ . For maximal mixing of  $\nu_\mu - \nu_\tau$ , as needed to fit the atmospheric neutrino data, this gives  $\sin^2 2\theta_{e\mu} \cong 8 \times 10^{-3}$ , which is in the allowed range for small-angle MSW.

On the other hand, if one assumes that  $M_R$  has large 12, 21 and/or 13, 31 elements, something quite remarkable happens. To illustrate, consider the form

$$M_R = \begin{pmatrix} 0 & D\epsilon^3 & 0 \\ D\epsilon^3 & B\epsilon^2 & 0 \\ 0 & 0 & A \end{pmatrix} m_R, \quad (13)$$

where we parameterize using powers of  $\epsilon$  merely for convenience. The light neutrino mass matrix comes out to be

$$M_\nu = -N^T M_R^{-1} N = \begin{pmatrix} \frac{\eta^2 AB}{\epsilon^4 D^2} & 0 & -\frac{\eta A}{\epsilon^2 D} \\ 0 & \epsilon^2 & \epsilon \\ -\frac{\eta A}{\epsilon^2 D} & \epsilon & 1 \end{pmatrix} \frac{m_U^2}{Am_R}. \quad (14)$$

One sees that the 2-3 block has vanishing determinant, so that a rotation in the 2-3 plane by an angle  $\theta_{23}^\nu \cong \epsilon$  brings  $M_\nu$  to the form



$$M'_\nu \cong \begin{pmatrix} \frac{\eta^2}{\epsilon^4} \frac{AB}{D^2} & \frac{\eta}{\epsilon} \frac{A}{D} & -\frac{\eta}{\epsilon^2} \frac{A}{D} \\ \frac{\eta}{\epsilon} \frac{A}{D} & 0 & 0 \\ -\frac{\eta}{\epsilon^2} \frac{A}{D} & 0 & 1 \end{pmatrix} \frac{m_U^2}{Am_R}. \quad (15)$$

The important thing to notice is that the 1-2 block has a pseudo-Dirac form. What happens, as a result of this, is that there is almost exactly maximal mixing between the electron neutrino and the muon neutrino. In fact, one has “bimaximal” mixing. Interestingly, the large value of  $U_{\mu 3}$  arises, as we have said, from the *charged* lepton mass matrix, whereas the large value of  $U_{e2}$  is coming, as we have just seen, from the *neutrino* mass matrix. When the parameter values are looked at more closely, it is found that the vacuum solution is easily obtained, but the large-angle MSW solution requires some fine-tuning of parameters.

(9) Finally, there is a prediction for the  $\nu_e - \nu_\tau$  mixing angle. It is easy to show that in *both* cases, the small-angle MSW case and the bimaximal vacuum oscillation case, the mixing  $U_{e3}$  comes out the same:  $U_{e3} \cong 0.07 \sin \theta_{\mu\tau}$ .

In conclusion, we have shown that a very simple explanation exists based on the group theory of  $SU(5)$  that accounts for the fact that the mixing of the left-handed fermions of the second and third families is small for quarks ( $V_{cb} \cong 0.04$ ) and large for leptons ( $\sin^2 2\theta_{\mu\tau} \cong 1$ ). We also showed how this explanation arose from a particular  $SO(10)$  model of fermion masses. This model was seen to be very simple, both at the level of the underlying  $SO(10)$  structures, and at the level of the resulting mass matrices. Because these matrices involve few parameters, no fewer than nine predictions result. Several of these will provide very non-trivial tests of the model, in particular the predictions for  $V_{ub}$  and the mixings  $U_{e2}$  and  $U_{e3}$ . For further details the reader can consult the series of papers in Refs 4 and 5.

## References

1. Weinberg, S. *Trans. N.Y. Acad. Sci.* **38**, 185 (1977); Wilczek, F. and Zee, A. *Phys. Lett.* **B70**, 418 (1977); Fritzsche, H. *Phys. Lett.* **B70**, 436 (1977).
2. Fritzsche, H. *Phys. Lett.* **73B**, 317 (1978).
3. Albright, C.H. and Barr, S.M. *Phys. Rev.* **D58**, 013002 (1998). (9712488)
4. Albright, C.H., Babu, K.S. and Barr, S.M. *Phys. Rev. Lett.* **81**, 1167 (1998) (hep-ph/9802314). Albright, C.H. and Barr, S.M. *Phys. Lett.* **B452**, 287 (1999).

5. Albright, C.H. and Barr, S.M. *Phys. Lett.* **B461**, 218 (1999).
6. Sato, J. and Yanagida, T. *Phys. Lett.* **B430**, 127 (1998). (hep-ph/9710516)
7. Irges, N., Lavignac, S. and Ramond, P. *Phys. Rev.* **D58**, 035003 (1998). (hep-ph/9802334)
8. Hagiwara, K. and Okamura, N. *Nucl. Phys.* **B548**, 60 (1999) (hep-ph/9811495).
- Altarelli, G. and Feruglio, F. *Phys. Lett.* **B451**, 388 (1999) (hep-ph/9812475).
- Berezhiani, Z. and Rossi, A. hep-ph/9907397.
9. Babu, K.S., Pati, J. and Wilczek, F. hep-ph/9812538.
10. Barr, S.M. and Raby, S. *Phys. Rev. Lett.* **79**, 4748 (1997).